THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL
ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

142/1 ADVANCED MATHEMATICS 1
(For Both School and Private Candidates)

Time: 3 Hours **2007 February, 13 Tuesday a.m.**

INSTRUCTIONS

1. This paper consists of sections A and B.
2. Answer all questions in section A and four (4) questions from section B.
3. All work done in answering each question must be clearly shown.
4. Mathematical tables, mathematical formulae, slide rules and non-programmable scientific calculators may be used.
5. Cellular phones are **not** allowed in the examination room.
6. Write your Examination Number on every page of your answer booklet (s).

SECTION A (60 marks)

Answer all questions in this section showing all necessary steps and answers.

1. (a) Simplify the set expression $[A^c \cap (A^c \cap B^c)]^c$ using the laws of algebra of sets. **(2 marks)**

(b) Sets A, B and C are defined as follows:

$A = \{x \in R : x < -1 \text{ or } x \geq 2\}$;

$B = \{x \in R : |x| \geq 2\}$ and ,

$C = \{x \in R : 1 \leq x \leq 4\}$

Sketch on the number line the following sets:

(i) $(A - B) \cup C$.

(ii) $(A \cup B) \cap C$. **(4 marks)**

2. (a) The straight line $2y - x - 16 = 0$ is a perpendicular bisector of the line joining the points A and B. If A is the point (-3, 4); determine the coordinates of B. **(3 marks)**

(b) P moves so that its distance from the origin is always equals to the shortest distance from the line $x = 5$. Find the equation of the locus. **(3 marks)**

3. (a) By using the functions $f(x) = \frac{x}{1+x}$, $g(x) = x^2$ and $h(x) = \frac{1}{x}$, where $x \neq 0$ or $x \neq -1$; show that function composition is

(i) associative.

(ii) not commutative . **(3 marks)**

(b) Given that $f(x) = 10x$ and $g(x) = x + 3$, show that $(fog)^{-1}(x) = g^{-1} of^{-1}(x)$. **(3 marks)**

4. (a) Find the value of x given that:

$$\log_2(2x+1) = \frac{4\log_{x-3} 2 + 1}{\log_{x-3} \frac{1}{2}}$$ **(3 marks)**

(b) Find the sum of series

$$\frac{5}{1 \times 2 \times 3} + \frac{8}{2 \times 3 \times 4} + \frac{11}{3 \times 4 \times 5} \ldots + \frac{3n+2}{n(n+1)(n+2)}.$$ **(3 marks)**

5. (a) Show that in ΔABC, $b + c = a\text{Cos}\frac{1}{2}(B - C)\text{Cosec}\frac{1}{2}A$. **(3 marks)**

(b) Without using tables, find the value of $\tan\left(\text{Cos}^{-1}\left(\frac{1}{2}\right) - \tan^{-1}\left(\frac{\sqrt{3}}{3}\right)\right)$ **(3 marks)**

6. (a) Differentiate $f(x) = \frac{1}{1+x}$ from first principles. **(3 marks)**

(b) The curve is defined parametrically as

$x = 2t^2 + 5t + 1$

$y = t + 6$ Find its gradient at $t = 1$ **(3 marks)**

7. (a) Find the cosine of the angle between $\underset{\sim}{BA}$ and $\underset{\sim}{BC}$ where A, B and C are the points (0,1,3), (-1,0,1) and C (1,-1,-2) respectively. **(3 marks)**

(b) Under the action of the forces $\underset{\sim}{F_1} = (-\underset{\sim}{i} + \underset{\sim}{j} + 2\underset{\sim}{k})$ N and $\underset{\sim}{F_2} = (-4\underset{\sim}{i} + 6\underset{\sim}{j} + 2\underset{\sim}{k})$ N, the body moved a distance $\sqrt{19}$ metres in the direction of the resultant force. Compute the work done on the body correct to two decimal places. **(3 marks)**

8. (a) Find: (i) $\int \frac{dx}{(x+1)\sqrt{x^2-1}}$ **(2 marks)**

(ii) $\int \frac{x+3}{x^3 - x^2 - 2x} dx$ **(2 marks)**

(b) Evaluate $\int_0^{\frac{\pi}{2}} \text{Cos}^5 x \, \text{Sin}^2 x \, dx$ **(2 marks)**

9. (a) Team A has probability $\frac{2}{3}$ of winning wherever it plays. If A plays 4 games, find the probability that A wins:

(i) at least 1 game.

(ii) more than half of the games. **(3 marks)**

(b) Cherry and Passion are in the table tennis tournament such that the first to win three games wins the tournament. By using tree diagram, how many logic possibilities of the tournament will occur? **(3 marks)**

10. Below is a frequency distribution table showing the marks obtained by 130 candidates in two different subjects, A and B.

Percentage	1-10	11-20	21-30	31-40	41-50	51-60	61-70	71-80	81-90	91-100
A	0	0	1	3	6	24	30	31	22	13
B	5	26	30	28	25	9	5	0	1	1

(a) Construct a table showing the cumulative frequency distribution in each subject and draw in one diagram the graphs of their ogives. **(4 marks)**

(b) From the diagram in (a) above, determine the percentage number of candidates that fail in each subject if the pass mark in subject A is 55 and that in subject B is 35. **(2 marks)**

SECTION B (40 marks)

Answer four (4) questions from this section, showing all necessary steps and answers.

11. In a certain garage, the manager had the following facts: floor space required for a saloon car is 2 m^2 and for a lorry is 3 m^2. Four technicians are required to service a saloon car and three technicians for a lorry per day. He has a maximum of 24 m^2 of floor space and a maximum of 36 technicians available. In addition he is not allowed to service more lorries than saloon cars. The profit for servicing a saloon car is 40,000/= and a lorry is 60,000/=. How many motor vehicles of each type should be serviced daily in order to maximize the profit? **(10 marks)**

12. (a) Find $\bar{W}$ and $\bar{Z}$ if W and Z satisfied the system $\left.\begin{aligned} 3Z + iW &= 2 \\ iZ - 2W &= -3i \end{aligned}\right\}$ **(4 marks)**

(b) Express $Z = 1 - i$ in polar form. Hence find the two complex values of W if $W^2 = Z$ leaving your answer in polar form. **(4 marks)**

(c) Write the complex number $\dfrac{3-i}{1+2i}$ in polynomial form. **(2 marks)**

13. (a) Show that the equation $e^x = 3 - x$ has roots in the interval [0,1]. Also, find this root correct to two decimal places in three iterations by using Regular Falsi method. **(6 marks)**

(b) Use trapezoidal rule and nine ordinates to obtain an approximate value of the definite integral.

$$\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}\,dx.$$ **(4 marks)**

14. (a) Solve the differential equation $\cot x \dfrac{dy}{dx} = 1 - y^2$ given that $y = 0$ when $x = \dfrac{\pi}{4}$. **(4 marks)**

(b) The rate from which the atoms in a mass of radioactive material are disintegrating is proportional to N, the number of atoms present at any time. Initially the number of atoms was M.

(i) Form and solve the differential equation that represents this data.
(ii) Given that half of the original mass disintegrates in 152 days, evaluate the constant of proportionality in the differential equation.
(iii) Sketch the graph to represent the number of atoms N at any time t. **(6 marks)**

15. (a) Show that $\text{Var}(x) = E(x^2) - [E(x)]^2$ **(3 marks)**

(b) The amount of sulphur oxide produced by an industrial plant in 80 days is as shown in the following table.

Tons of Sulphur Oxide	5.0-8.9	9.0-12.9	13.0-16.9	17.0-20.9	21.0-24.9	25.0-28.9	29.0-32.9
Frequency	3	10	14	25	17	9	2

Calculate the interquartile range for this distribution. **(7 marks)**

16. (a) Three forces are exerted on an object as follows:

$\underline{F}_1 = 5$ units to the right; $\underline{F}_2 = 10$ units upwards and $\underline{F}_3 = 2$ units inclined at an angle of 30° to the horizontal in the upward direction.
Find in terms of i and j a single force equal to the three forces acting together. **(3 marks)**

(b) A mouth of the gun is inclined at an angle of 30° to the horizontal. The gun is fired from ground level with an initial speed of 1500 m/s. Assuming that the gun is fired at the origin of the xy plane, determine the equation for the path of the bullet. (Take $g = 10\ m/s^2$). **(7 marks)**