THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL OF TANZANIA
ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

142/1

ADVANCED MATHEMATICS 1
(For Both School and Private Candidates)

Time: 3 Hours **Tuesday, 02nd May 2017 a.m.**

Instructions

1. This paper consists of **ten (10)** questions, each carrying **ten (10)** marks.
2. Answer **all** questions.
3. **All** necessary working and answers for each question done must be shown clearly.
4. Mathematical tables and non-programmable calculators may be used.
5. Cellular phones are **not** allowed in the examination room.
6. Write your **Examination Number** on every page of your answer booklet(s).

1. (a) By using a scientific calculator compute:

(i) $\dfrac{\sqrt{240}\times e^{\ln\frac{1}{3}}\sin 22^\circ}{\sqrt{\tan 17^\circ}\times 3^{4\ln 11}}$ correct to 3 significant figures,

(ii) $\ln\dfrac{\sqrt{98.2}\times(0.0076)^{-1}\times 10^{7}}{\tan\dfrac{\pi}{3}\times\cos^{3}\dfrac{\pi}{4}}$ correct to 6 significant figures,

(iii) $\sqrt{\dfrac{(0.485)^{6}+\tan^{-1}(1.54)e}{(62.54)^{4}\sin^{-1}(0.4561)}}$ correct to 4 decimal places.

(b) If $M^{d}=\dfrac{p}{\pi t^{2}}\left[\dfrac{4}{3}\ln\left(\dfrac{D}{d}\right)+\sqrt{\log p}\right]^{\frac{1}{3}}$, with the aid of a non programmable calculator evaluate D given that $P=1.6\times10^{3}$, $t=56\times10^{-2}$, $M=50.6\times10^{2}$ and $d=\lim\limits_{x\to\infty}\left(\dfrac{\cosh x}{e^{x}}\right)$ correct to four decimal places.

2. (a) If $x=\ln\left\{\tan\left(\dfrac{\pi}{4}+\dfrac{\theta}{2}\right)\right\}$, find e^{x} and e^{-x} and hence show that $\sinh x=\tan\theta$.

(b) If $a\cosh x+b\sinh x=c$, show that the value of $x=\ln\left(\dfrac{c\pm\sqrt{c^{2}+b^{2}-a^{2}}}{a+b}\right)$.

(c) Use the appropriate hyperbolic substitution to evaluate $\int_{0.1}^{0.8}\sqrt{x^{2}+4x+3}\,dx$.

3. Following an illness, a patient is required to take pills containing minerals and vitamins. The contents and costs of two types of pills, Felgood and Getbetta, together with the patient's daily requirement, are shown in the following table:

	Mineral	**Vitamin**	**Cost**
Feelgood	80 mg	4 mg	3,000/=
Getbetta	20 mg	3 mg	1,500/=
Daily requirement	420 mg	31 mg	

If the daily prescription contains x Feelgood pills and y Getbetta pills, find the cheapest way of prescribing the pills and the cost.

4. The following table shows distribution of marks in a matriculation examination of communication skills:

Marks	11-20	21-30	31-40	41-50	51-60	61-70	71-80	81-90	91-100
Frequency	8	12	18	25	40	28	31	30	8

(i) Given that the assumed mean is 75.5, use the coding method to find the average marks.

(ii) Determine the lower quartile of the distribution.

(iii) Calculate the 75^{th} percentile correct to four significant figures.

5. (a) Use the laws of algebra to simplify:

(i) $[A \cap (B \cap C')] \cup C$.

(ii) $(X \cap Y') \cup (X \cap Y) \cup (Y \cap X')$.

(b) Out of a group of 17 girl guides and 15 boy scouts, 22 play handball, 16 play basketball, 12 of the boy scouts play handball, 11 of the boy scouts play basketball, 10 of the boy scouts play both and 3 of the girl play neither of the two.

(i) How many girls play both handball and basketball?

(ii) How many in the group play handball only and basketball only?

6. (a) Draw the graph of $f(x) = x^3 - 3x^2 - 6x + 8$ in the interval $[-5, 6]$. Hence tell how $f(x)$ behaves for positively and negatively large values of x.

(b) Find $f \circ g(x)$ given that $f(x) = 2x^2 + 1$ and $g(x) = \dfrac{4x}{x^2 - 2}$, hence

(i) Determine the vertical and horizontal asymptotes of $fog(x)$.

(ii) Draw the graph of $f \circ g(x)$.

(iii) State the domain and range of $f \circ g(x)$.

7. (a) Show that the Newton Raphson Formula of finding the roots of the equation $12x^3 + 4x^2 - 15x - 4 = 0$ is $x_{n+1} = \dfrac{(24x_n + 4)x_n^2 + 4}{(36x_n + 8)x_n - 15}$ and use this formula to find the roots of $12x^3 + 4x^2 - 15x - 4 = 0$ correct to three decimal places.

(b) Approximate the area under the curve $y = \dfrac{1}{x - 2}$ between $x = 2$ and $x = 3$ with six ordinates by:

(i) Trapezoidal rule,

(ii) Simpson rule.

(c) Which among the rules in 7(b) gives a better approximation to the area?

8. (a) Find the value of k such that $k(x^2 + y^2) + (y - 2x + 1)(y + 2x + 3) = 0$ is a circle. Hence obtain the centre and radius of the resulting circle.

(b) The circle $x^2 + y^2 - 2x - 4y - 5 = 0$ has a centre C and is cut by the line $y = 2x + 5$ at A and B. Show that BC is perpendicular to AC and hence find the area of triangle ABC.

(c) Find the equation of the straight line which passes through the intersection of the lines $3x + 2y + 4 = 0$ and $x - y = 2$ and forms the triangle with the axes whose area is 8 square units.

9. (a) Evaluate $I_{ab} = \int \sin ax \cos bx \, dx$ if $a \neq b$ and use it to find the value of n in $\int_0^n \sin 3x \cos 2x \, dx = \frac{3 - \sqrt{3}}{5}$.

(b) Find the length of arc of the semi-cubical parabola $y^2 = x^3$ between the points $(1, 1)$ and $(4, 8)$.

10. (a) If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, prove that $\frac{dy}{dx} = -\frac{1}{(1+x)^2}$.

(b) Given that $f = \sin xy$, find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$

(c) Using Taylor's theorem, expand $\sin\left(\frac{\pi}{6} + h\right)$ in ascending power of h up to the h^4 term and hence evaluate $\sin 31°$ correct to three decimal places.