ADVANCED MATHEMATICS
LOGIC

Reasoning

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Publisher: Baraka Loibanguti

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To my wife Caroline

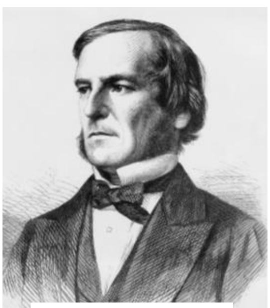
LOGIC

Chapter

3

The rules of logic give precise measuring to mathematical statements. These rules are used to distinguish between valid and invalid mathematical arguments.

In addition, it is importance in understanding mathematical reasoning; logic has numerous applications in computer science. These rules are used in design of computer circuits the construction of computer programs, the verification of the correctness of programs and in many other ways.



G.L. Boolean

Logic is all about reasons. Every day we consider possibilities, we think about what follows from different assumptions, what would be the case in different alternatives, and we weight up competing positions or options. In all of this, we reason. Logic is the study of good reasoning, and in particular, what makes good reasoning good.

To understand good reasoning, we must have an idea of the kinds of things we reason about. What are the things we give reasons for? We can give reasons for doing something rather than something else (these are reasons for actions) or for liking some things above other things, (these are reasons for preferences). In the study of logic, we do not so much look at these kinds of reasoning: instead, logic concerns itself with reasons for believing something instead of something else. For beliefs are special, it includes faith. They function not only as the outcome of reasoning, but also as the premises in our reasoning. So, we start with the following question: What are the sorts of things we believe? What are the things we reason with?

PROPOSITIONS

A proposition is a statement that is either true or false but not both or neither. Proposition are denoted by letters just as letters used in algebra. The most common letter used to represent propositions are p , q and r although any other letter may be used.

Sentence is a group of words to give out the meaning phrase. This book is not discussing more about sentences but propositions (statements).

Example of propositions

(a) Arusha is a capital city of Tanzania
(b) You are a single girl.
(c) Dar Es Salaam is the city in Eastern Tanzania
(d) Kampala is a capital city of Kenya
(e) $3 + 5 = 8$
(f) $2 + 2 = 22$

The propositions (a), (d) and (f) are false whereas propositions (c) and (e) are true statements, (b) is neither true nor false, it is hard to tell (this is not a proposition). Each sentence in this list is the kind of thing to which we might assent or dissent. We may agree or disagree, believe or disbelieve, or simply be undecided, about each of these claims.

Not every sentence is proposition; Sentences such as those expressing emotions, greeting, asking questions or making requests are not propositions.

Example of sentences, which are not propositions: Questions, emotion expression, commands, etc.

(a) What is your name?
(b) Hello! How are you?
(c) Where is my book?
(d) $3x + 1 = 7$
(e) $x + y = z$
(f) I am very happy today!

All these are not propositioning. Propositions are the sorts of things that can be true or false. In the study of logic, we are interested in relationships between propositions, and the way in which some propositions can act as reasons for other propositions. These reasons are what we call *arguments.*

An argument is a list of propositions, called the premises, followed by a word such as 'therefore', or 'so', and then another proposition, called the conclusion.

Example of an argument.

"If I study Mathematics, I will not study physics. I am studying physics; therefore, I am not studying Mathematics"

The first two sentences

(a) *If I study Mathematics, I will not study Physics.*
(b) *I am studying Physics.*

Are premises, while the last sentence

(c) *Therefore, I am not studying Mathematics.*

Is a conclusion

For an argument to be good is for the premises to guarantee the conclusion. That is if the premises are true then the conclusion has to be true. These good arguments are called **valid arguments.** Generally, an argument is valid if and only if whenever the premises are true, so is the conclusion. In other words, it is impossible for the premises to be true while at the same time the conclusion is false.

Consider the next argument below

"*If people are mammals, they are not cold-blooded. People are cold-blooded. Therefore, people are not mammals*"

Note that, the argument may be invalid in real life but valid logically. Example of the argument that is invalid in real life but valid logically

"If human are cold blooded animals, then they are reptilians, human are cold blooded animas. Therefore, they are reptilians".

This argument is invalid in real world; we need to be careful here because logically this argument is **valid**.

This is clearly a valid argument, but no one in his or her right mind would believe the conclusion. That is because the premises are false. Well, the second premise is false—but that is enough. One premise being false is enough for the argument to be bad. The bad arguments are called **invalid arguments.**

Sometimes we do not have enough information in our premises to guarantee the conclusion, but we might make the conclusion more likely than it might be otherwise. We say that an argument is inductively strong if, given the truth of the premises, the conclusion is likely. An argument is inductively weak if the truth of the premises does not make the conclusion likely. The study of inductive strength and weakness is the study of inductive logic. We will not explore inductive logic in this book. We will concentrate on what is called deductive logic — the study of validity of arguments.

Consider the argument below once again

If people are mammals, they are not cold-blooded. People are cold-blooded.

Let p be "*if people are mammals*" and q be "*they are cold blooded*"

These premises have the following structure

"If p then not q, therefore not p"

- *Be warned*—an argument can be an instance of an invalid form, while still being valid.

Logic is all about reasoning. Letters are used to denote propositions just as letters used to denote variables in algebra.

From the book of Thought written by English Mathematician *George Boole 1954,* we have methods of producing new propositions from those that we already have. Many Mathematical statements are constructed by combining one or more propositions.

Definition: *"Compound proposition is formed from existing proposition using logical operators. These are mathematical statements constructed by combining two or more propositions"*

TRUTH TABLE

The truth table displays the relationship between the truth-values of proposition. The truth table are especially valuable in the determination of the truth-values of propositions constructed from simpler propositions. To construct the truth table, follow steps below: -

(i) First column has $T=2^{n-1}$ and $F=2^{n-1}$ truth values

(ii) The second column has $T=2^{n-2}$ and $F=2^{n-2}$ truth values

(iii) The r^{th} column has $T=2^{2-r}$ and $F=2^{2-r}$ truth-values, where n is number of simpler propositions and r is the specified column. Numbers of T and F occurs alternate in the truth table.

LOGIC OPERATORS

The propositional connectives are *negation* ($\neg$, not), conjunction ($\wedge$, and), disjunction ($\vee$, or), implication ($\Rightarrow$, if - then) and bi-implication ($\Leftrightarrow$, if and only if). The connectives $\wedge$, $\vee$, $\Rightarrow$ and $\Leftrightarrow$ are designated as binary while $\neg$ is designated as unary.

Conjunction (and, $\wedge$)

Let p and q be simple propositions. The proposition "p and q" denoted by $p \wedge q$ is the proposition that is true when both p and q are true and is false otherwise. The proposition $p \wedge q$ is called the conjunction of p and q.

Here we have two propositions p and q therefore $n=2$ (number of simple propositions p and q)

Therefore T = 2 and F = 2 in the first column

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction (OR, $\vee$)

Let *p* and *q* be simple propositions. The proposition "*p* or *q*" denoted by $p \vee q$ is the proposition that is false when both *p* and *q* are false and is true otherwise. The proposition $p \vee q$ is called the disjunction of *p* and *q*.

Truth table of $p \vee q$ is given below

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Exclusive Or ($\oplus$)

Let *p* and *q* be propositions. The exclusive or of *p* and *q* is denoted by $p \oplus q$ is the proposition that is true when exactly one *p* and *q* is true and is false otherwise. The proposition $p \oplus q$ is called the **exclusive or** of p and q.

Truth table of $p \oplus q$ is given below

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Negation, (NOT, $\neg$)

Let *p* be a proposition. The negation of *p* is denoted by $\neg p$ or ~p. $\neg p$ is false only when *p* is true and false otherwise.

Truth table of $\neg p$

p	$\neg p$
T	F
T	F
F	T
F	T

Condition/Implication, ($\rightarrow$)

Let *p* and *q* be propositions. The implication $p \rightarrow q$ is the proposition that is false when *p* is true and *q* is false and is true otherwise. In this implication *p* is called **hypotheses, antecedent, or premise** and *q* is called the **conclusion** or **consequence.**

Truth table of $p \rightarrow q$

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Because implication arise in many places in mathematical reasoning a wide variety of terminology is used to express $p \rightarrow q$. Keywords used to show implication are:

(a) If p then q
(b) p is sufficient for q
(c) if p, q
(d) p only if q
(e) p is necessary for q
(f) q if p
(g) q whenever p

Note that $p \rightarrow q$ is false only in the case that *p* is true but *q* is false so that it is true when both *p* and *q* are true and when *p* is false (No matter what truth-value *q* has).

Consider the example below about implication "*If you make more than Tshs. 75,000/- per month then you must file a tax return*". This statement is saying nothing about somebody making less than Tshs. 75,000/- and do not file a tax return.

Similarly, the statement "*If a player hits more than 60 home runs, then a bonus of 10 million is awarded*", then this contract will be violated only if the player hits more than 60 home runs and not awarded but it says nothing if the player hits fewer than 60 home runs.

CONVERSE, INVERSE AND CONTRAPOSITIVE

Let *p* and *q* be two propositions. There are some related implications that can be formed from $p \rightarrow q$. The proposition $q \rightarrow p$ is called the **converse** of $p \rightarrow q$ and the proposition $\neg q \rightarrow \neg p$ is called **contrapositive** of $p \rightarrow q$. Simply, contrapositive is the inverse of converse.

We can build up compound propositions using the negation operator and the different connectives defined so far. Parentheses are used to specify the order in which the various logical operator in compound propositions are applied. In particular, the logical operator in the innermost parentheses are applied first.

Example 1

Write down the converse and contrapositive of the implication *"If today is Thursday, then I have a test today"*

Solution

Let p be "*today is Thursday*" and q *be "I have a test today"*, then the compound statement can be written as $p \to q$, it is converse $q \to p$, In words is *"If I have a test today, then today is Thursday"*. Contrapositive of $p \to q$ is $\neg q \to \neg p$ then it is *"If I do not have a test today, then today is not Thursday"*.

Biconditional/double implication, ($\leftrightarrow$)

Let p and q be propositions. The biconditional $p \leftrightarrow q$ is the proposition that is true when p and q have the same truth-values and false otherwise.

Truth table for $p \leftrightarrow q$

p	q	p↔q
T	T	T
T	F	F
F	T	F
F	F	T

Keywords to show biconditional

(a) "*p* if and only if *q*"

(b) "*p* is necessary and sufficient for *q*"

(c) "If *p* then *q* and conversely".

LAWS OF ALGEBRA OF PROPOSITION

Let *p, q* and *r* be the logical statements, where T and F stands for True and False values

Note that, the symbol $\Leftrightarrow$ show "*Logical equivalent*"

(a) Identity law

$p \wedge T \Leftrightarrow p$

$p \vee F \Leftrightarrow p$

$p \vee \neg p \Leftrightarrow T$

$p \wedge \neg p \Leftrightarrow F$

(b) Domination law

$p \vee T \Leftrightarrow T$

$p \wedge F \Leftrightarrow F$

(c) Idempotent law

$p \wedge p \Leftrightarrow p$

$p \vee p \Leftrightarrow p$

(d) Double negation law

$\neg(\neg p) \Leftrightarrow p$

(e) Commutative law

$p \wedge q \Leftrightarrow q \wedge p$

$p \vee q \Leftrightarrow q \vee p$

(f) Associative law

$(p \vee q) \vee r \Leftrightarrow p \vee (q \vee r)$

$(p \wedge q) \wedge r \Leftrightarrow p \wedge (q \wedge r)$

(g) Distributive law

$p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$

$p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$

(h) De Morgan's law

$\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$

$\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$

(i) $p \rightarrow q \Leftrightarrow \neg p \vee q$

$p \leftrightarrow q \Leftrightarrow (p \rightarrow q) \wedge (q \rightarrow p) \Leftrightarrow (\neg p \vee q) \wedge (\neg q \vee p)$

EXERCISE 1: LOGIC

1. Draw the truth table of the following
   (a) $(p \vee q) \rightarrow q$
   (b) $(p \rightarrow q) \rightarrow \neg p$
   (c) $(p \vee q) \rightarrow \neg r$
   (d) $(p \vee q) \rightarrow (p \rightarrow \neg q)$
   (e) $(p \rightarrow \neg q) \leftrightarrow \neg p$
   (f) $(p \leftrightarrow q) \rightarrow r \vee (q \rightarrow p)$
   (g) $(p \wedge q) \leftrightarrow (\neg p \vee q)$
2. Use the algebra laws of proposition simplify the following
   (a) $(p \vee q) \rightarrow [(p \wedge q) \leftrightarrow \neg q]$
   (b) $(\neg p \vee q) \leftrightarrow (\neg q \rightarrow p)$
   (c) $(\neg p \rightarrow q) \rightarrow (p \oplus q)$
   (d) $(q \leftrightarrow p) \wedge (\neg q \leftrightarrow p)$
   (e) $(p \leftrightarrow q) \rightarrow (p \rightarrow q)$
   (f) $(p \rightarrow q) \rightarrow (\neg q \rightarrow \neg p)$
   (g) $(p \wedge \neg q) \vee (\neg p \wedge q) \vee (p \vee q)$

ARGUMENTS

In everyday situations, arguments are dialogues between people. Here we do not study everything of these dialogues. We just stick on the propositions people express when they give reasons for things. In logic, argument is a list of propositions called the premises, we saw early, followed by a word such as "therefore" or "so" and then another proposition called conclusion.

In simple words; an argument is a sequence of statements.

Premises or assumptions or hypothesis these are statements in an argument.

Conclusion is the final statement in an argument.

Example 2

"If I live in Arusha, then I'm a Maasai. I'm a Maasai; therefore, I live in Arusha".

Solution

Let *p* be *"I live in Arusha*" and *q* be *"I am a Maasai"*

Here the conclusion is *"I live in Arusha"*

Logically equivalent

The compound propositions that have the same truth-values in all possible cases are called logically equivalent. The symbol $\Leftrightarrow$ stands for "Equivalent to".

One the way to show that the logical propositions are equivalent is by using truth table and another way is to use laws of algebra of propositions to resemble either left hand sides to the right-hand side or vice versa.

Propositions *p* and *q* are said to be logically equivalent if and only if the columns giving their truth-values agree otherwise, they are not logically equivalent.

Example 3

Use the truth table to show that $\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$

Solution

p	q	$\neg(p \vee q)$	$\neg p \wedge \neg q$
T	T	F	F
T	F	F	F
F	T	F	F
F	F	T	T

Because the columns of $\neg(p \vee q)$ and $\neg p \wedge \neg q$ on the truth table have the same truth values, then they are equivalent, written as $\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$

Example 4

Show that the propositions $p \rightarrow q$ and $\neg p \vee q$ are logically equivalent.

Solution

Using the truth table

p	q	$\neg p \vee q$	$p \rightarrow q$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

The columns of $\neg p \vee q$ and $p \rightarrow q$ have the same truth values, therefore $p \rightarrow q \Leftrightarrow \neg p \vee q$

TAUTOLOGY, FALLACY AND CONTINGENCY

Tautology is a compound proposition that is always true no matter what the truth-values of the proposition that occur in it.
In other words, Is the proposition which is true (T) in every row of the truth table.

Example 6

Use the truth table to show that $p \vee \neg p$ is tautology

Solution

p	$\neg p$	$p \vee \neg p$
T	F	T
T	F	T
F	T	T
F	T	T

Fallacy or Contradiction is a compound statement/proposition that is always false. It is the opposite of tautology.

Example of a contradiction proposition is $p \wedge \neg p$

p	$\neg p$	$p \wedge \neg p$
T	F	F
T	F	F
F	T	F
F	T	F

Contingency or invalid proposition: is a compound proposition, which is neither tautology nor fallacy.
Example of an invalid proposition is $(p \vee q) \rightarrow \neg p$

p	q	$p \vee q$	$(p \vee q) \rightarrow \neg p$
T	T	T	T
T	F	T	T
F	T	T	F
F	F	F	T

Because the last column $(p \vee q) \rightarrow \neg p$ has truth-values T and F then statement is nether tautology nor fallacy. This is contingency or invalid statement.

Example 7

Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are equivalent

Solution

p	q	$\neg(p \vee (\neg p \wedge q))$	$\neg p \wedge \neg q$
T	T	F	F
T	F	F	F
F	T	F	F
F	F	T	T

Hence $\neg(p \vee (\neg p \wedge q)) \Leftrightarrow \neg p \wedge \neg q$

Example 8

Show that $(p \wedge q) \rightarrow (p \vee q)$ is tautology using laws of proposition.

Solution

Given $(p \wedge q) \rightarrow (p \vee q)$

$\Leftrightarrow \neg(p \wedge q) \vee (p \vee q)$ - Negation law

$\Leftrightarrow \neg p \vee \neg q \vee p \vee q$ - De Morgan's law

$\Leftrightarrow (\neg p \vee p) \vee (\neg q \vee q)$ - Associative law

$\Leftrightarrow T \vee T$ - Identity law

$\Leftrightarrow T$ - Identity law

Hence $(p \wedge q) \rightarrow (p \vee q)$ is tautology proposition

TESTING VALIDITY OF ARGUMENTS

Valid argument: In logic the property of an argument consisting in the fact that the truth of the premises logically guarantee the truth of the conclusion.

The following are steps to test the validity of an argument

(a) Express the compound statement (argument) as a logical symbol

(b) Use the truth table to check the validity of an argument or use laws of algebra of propositions to simplify the compound statement

(c) Write the conclusion

Example 9

Test the validity of the following argument using the truth table *"If I love Mathematics then I will practice Mathematics. I do not practice Mathematics; therefore, I hate Mathematics"*

Solution

Using the truth table

Let *p* be *"I love mathematics"* and *q* be *"I will practice Mathematics"*

Symbolize, $[(p \to q) \wedge \neg q] \to \neg p$

Truth table

p	q	$(p \to q)$	$(p \to q) \wedge \neg q$	$[(p \to q) \wedge \neg q] \to \neg p$
T	T	T	F	T
T	F	F	F	T
F	T	T	F	T
F	F	T	T	T

Therefore, the argument is valid (tautology)

Using laws of algebra of propositions

$[(p \to q) \wedge \neg q] \to \neg p$

$[(\neg p \vee q) \wedge \neg q] \to \neg p$ - Negation law

$[(\neg p \wedge \neg q) \vee (\neg q \wedge \neg q)] \to \neg p$ - Distributive law

$[(\neg p \wedge \neg q) \vee F] \to \neg p$ - Identity law

$(\neg p \wedge \neg q) \to \neg p$ - Identity law

$\neg(\neg p \wedge \neg q) \vee \neg p$ - Negation law

$(p \vee q) \vee \neg p$ - Associative law

$(p \vee \neg p) \vee q$ - Associative law

$T \vee q$ - Identity law

T

Hence, the proposition is valid

Example 10

Test the validity of the argument; *"If 6 is even then 2 divide 7, either 5 is not prime or 2 divide 7, but 5 is Prime. Therefore 6 is odd".*

Solution

Let *p* be *"6 is even"* *q* be *"2 divide 7"* and *r* be *"5 is prime"*

Symbolically

$$[(p \to q) \wedge (\neg r \vee q) \wedge r] \to \neg p$$

Using the truth table

p	q	r	$p \to q$	$\neg r \vee q$	$(p \to q) \wedge (\neg r \vee q) \wedge r$	$\left[(p \to q) \wedge (\neg r \vee q) \wedge r\right] \to \neg p$
T	T	T	T	F	F	T
T	T	F	T	T	F	T
T	F	T	F	F	F	T
T	F	F	F	T	F	T
F	T	T	T	T	T	T
F	T	F	T	F	F	T
F	F	T	T	T	T	T
F	F	F	T	F	F	T

Using the truth table, the conclusion is valid.

Example 11

Determine the validity of the following statement: *"If there is no Law, there is no justice. There is no Law, there is no justice"*

Solution

Let *p* be *"there is no law"* and *q* be *"there is no justice"*

Symbolically

$(p \to q) \wedge (p \to q)$ by the truth table

p	q	$p \to q$	$p \to q$	$(p \to q) \wedge (p \to q)$
T	T	T	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

The conclusion from the truth table shows that the statement is invalid

ACTIVITY 2

1. (a) Check the validity of the following argument $\neg p \to (q \leftrightarrow \neg r)$, $\neg r \to \neg p$, $q \to \neg r$ and $\neg r$

   (b) What does the proposition $[(p \wedge q) \wedge r] \to (p \wedge q)$ represent?

2. (a)What is mean by the terms;

   (i) Fallacy statement

   (ii) Tautology statement

   (b) Translate into symbolic form and test the validity of the following argument: "If Kanzi speaks logically, then He does not contradict laws of Algebra. Either there is no meaning or He contradicts laws of Algebra. However, there is meaning. Therefore, Kanzi speaks logically".

3. (a) Simplify the following $[(p \wedge q) \vee r] \to (p \wedge q)$

   (b) Draw the truth table of $\neg(p \wedge q) \vee p$ and conclude

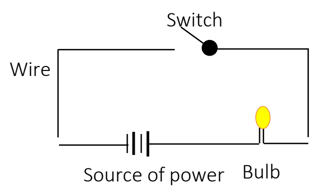
4. Symbolize the following argument and conclude "If I am headache, then I have malaria or low blood pressure, I have no malaria nor low blood pressure, therefore I am not headache".

5. Test the validity of the statements: If 3 + 6 = 9 then $3x + 2x = 6x$, therefore 4 + 5 = 11 or 9 – 5 = 4.

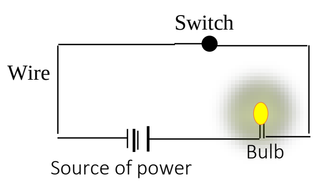
ELECTRICAL NETWORK

Logic is very useful in daily life of electric. For the bulb to be light up the complete circuit must be well connected, thus wire, source of power and bulb itself. Electric circuit can also be used to verify or test the validity of the logical statements (compound statements).

Consider the circuits below



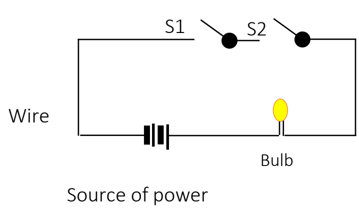
This circuit is open (off), the current is not passing therefore the bulb is not light up. When a switch closed (on) the current will pass to the bulb, which make it to light up.



This circuit is closed (on), the current is passing therefore the bulb is light up.

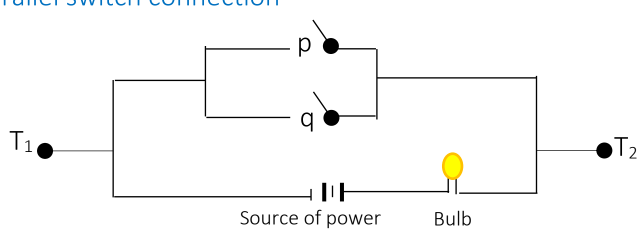
Parallel and series connection of switches

Series switch connection



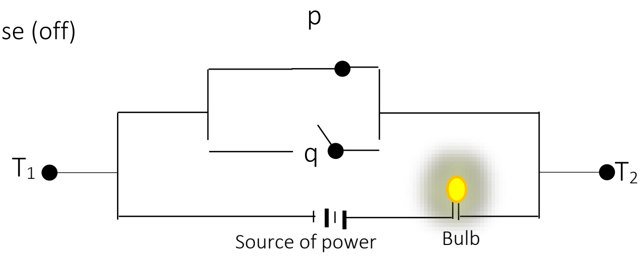
Switches S1 and S2 are series switches. When S1 switch is closed (on) and S2 is open (on) the bulb will not light up. For the bulb to light up both switches must be closed (on) otherwise the bulb will not work.

Parallel switch connection



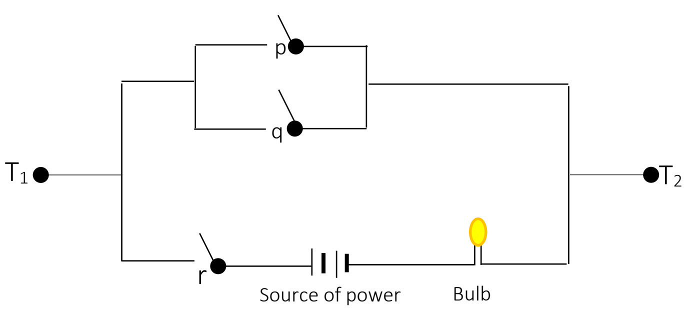
To light the bulb all one switch (p or q) need to be closed (on). For this case, the bulb will light when one or both (all) the parallel switches turned on.

Using the logic concept, one or both switches should be true (on) but not both for the bulb to work and it will not work only when both p and q are false (off)



As seen here only switch p is closed and q is open, this makes the current to pass through the switch p to connect the source of power and the bulb, which makes the bulb to light up.

Let p and q be the compound statement represented by the following circuits, the series switches are connected by the word "and" while the parallel switches are connected by the word "or".



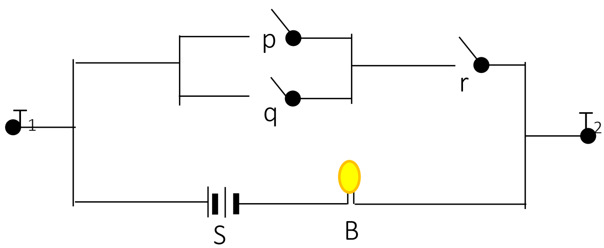
Switch p and q are parallel and both are in series with switch r. Even if switch p and q will be closed and r left, open there is no way a bulb will light up. Switch r controls current to both switch p and q, logically this can be expressed as $(p \vee q) \wedge r$.

If switch p and q are closed and switch r is open, still the bulb will not work, because r disconnect the current to reach the bulb.

Note that, when switches are opposite of the normal switch logically is interpreted as negation of the switch.

Example 12

Express the following electric circuit in logical symbols using the switch letters given



Solution

Switch p and q are parallel and r is in series with p and q, this can be written as $(p \vee q) \wedge r$

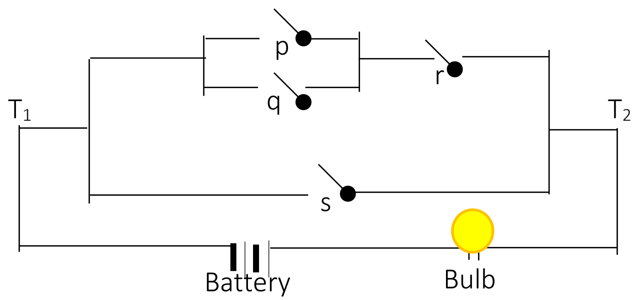
Example 13

Construct an electrical network corresponding to the proposition $[(p \vee q) \wedge r] \vee s$

Solution

p and q are parallel, $p \vee q$ is in series with r and lastly $(p \vee q) \wedge r$ is parallel to switch s

The circuit look as follows



MISCELLANEOUS EXERCISE – LOGIC

1. Use properties of operations in logic to show whether the following proposition is a tautology $[\neg(p \vee q) \vee (\neg p \wedge q)] \vee p$
2. What is a proposition as used in mathematics logic?
3. Show that $\neg(\neg p)$ and **p** are logically equivalent
4. Use the truth table to show that the following proposition are logically equivalent.
   a) $(p \vee q)$ and $(q \vee p)$
   b) $(p \wedge q)$ and $(q \wedge p)$
5. Verify the following associative laws by truth table $(p \vee q) \vee r = p \vee (q \vee r)$
6. Use the truth table very that $(p \wedge q) \Leftrightarrow (\neg p \vee \neg q)$
7. Prove that $p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$ truth table.
8. Determine the contrapositive of the statement: “If John is a jobless, then he is poor”
9. Show that each of the following are tautology using truth table
   a) $(p \wedge q) \rightarrow p$ b) $p \rightarrow (p \vee q)$ c) $\neg p \rightarrow (p \rightarrow q)$
10. Use laws of algebra of propositions show that the following are tautology
    a) $p \rightarrow (p \vee q)$ b) $\neg(p \rightarrow q) \rightarrow \neg q$ c) $\neg p \rightarrow (p \rightarrow q)$
    d) $(p \wedge q) \rightarrow (p \rightarrow q)$
11. Use laws of algebra simplify the following
    a) $[\neg p \wedge (p \vee q)] \rightarrow p$
    b) $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$
    c) $[p \wedge \neg(p \rightarrow q)] \rightarrow q$
    d) $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$
12. Verify the following absorption laws
    a) $[p \vee (p \wedge q)] \Leftrightarrow p$
    b) $[p \wedge (p \vee q)] \Leftrightarrow p$
13. Determine if the following propositions are tautology or not
    a) $\neg p \wedge (p \rightarrow q) \rightarrow \neg q$

b) $\neg q \wedge (p \to q) \to \neg p$

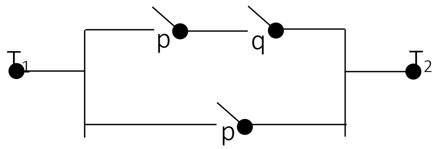
14. Show that $p \leftrightarrow q$ is equivalent to $(p \wedge q) \vee (\neg p \wedge \neg q)$
15. Determine if $(p \to q) \to r$ and $p \to (q \to r)$ are not equivalent.
16. Show that $p \to q$ is equivalent to it is contrapositive
17. Use truth table to show that $\neg p \leftrightarrow q$ and $p \leftrightarrow \neg q$ are logically equivalent
18. Show that $\neg(p \oplus q)$ and $p \leftrightarrow q$ are logically equivalent
19. Write the negation of the following statements
    (a) *If she buys book, she will read*
    (b) *If it rains, he will not go to play*
    (c) *If it is a sunny day, we will wash our clothes*
20. Symbolize the following statements and write their inverse, converse and contrapositive
    (a) *If Mathematics is interesting then I will not study Physics*
    (b) *If I get grade A in Physics then I will be an engineer*
    (c) *If 3 is even, then 4 is odd*
    (d) *History is interesting if and only if mathematics is boring and tough*
21. Construct the truth table for the following statements
    (a) *If I work hard, I will be rich*
    (b) *A number is even if and only if it is divisible by 2*
    (c) *You will pass your examinations if you study hard*
    (d) *We will play football if there is no rain or extensive sunlight*
    (e) *You like Geography and Advanced Mathematics or you do not like Geography and Economics'*
22. Draw a simple electric network diagram for the statements
    (a) $(p \to q) \wedge (p \vee q)$
    (b) $(p \wedge \neg q) \vee (\neg p \vee q)$
23. Prove that $(p \to q) \leftrightarrow p$ and $p \wedge q$ are equivalent, using
    (a) Laws of algebra of sets
    (b) The truth tables
24. Draw the electric network for the proposition $\neg(p \vee q) \vee (\neg p \wedge q)$
25. Use laws of algebra of sets to show if $[\neg(p \vee q) \vee (\neg p \wedge q)] \vee p$

26. Construct an electrical network corresponding to the proposition $[(p \vee q) \wedge r] \vee s$

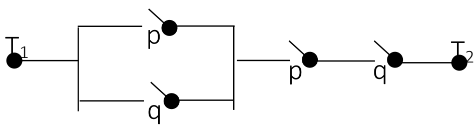
27. Differentiate between tautology, contradiction and contingency

28. Simplify the following electrical network and re-draw

(a)



(b)



29. Consider the truth table below

p	q	X	Y
T	T	F	F
T	F	T	F
F	T	F	T
F	F	F	F

Find the compound statements represented by letters X and Y and hence draw an electric network to represent the simplified propositions

30. Consider the truth table below

p	q	r	M	N	L	S
T	T	T	F	T	T	T
T	T	F	F	T	T	T
T	F	T	T	T	T	T
T	F	F	F	T	T	T
F	T	T	F	F	F	F
F	T	F	F	T	T	F
F	F	T	T	F	T	F
F	F	F	F	T	T	F

Find the propositions represented as the letters M, N, L and S and draw electric network for each statement above.

31. Without simplifying, the propositions $(p \vee \neg q) \wedge (r \vee q)$ draw an electric network and construct a truth table.

32. Define the following terminologies as used in logic
(a) Proposition
(b) Disjunction
(c) Contradiction

33. (a) Simply the compound statement $\neg[(p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q)]$ and an electric network.
(b) Using the letters and logical connectives, write the following statement *"If q is less than zero then is not positive"*
(c) Given the statement *"If q is less than zero then is not positive"* write: -
(i) The contrapositive of converse
(ii) Converse of the inverse
(iii) Contrapositive of inverse of converse

34. (a) Show whether $(p \Rightarrow q) \Rightarrow (q \Rightarrow r)$ is tautology, invalid or fallacy.
(b) Test the validity of this compound statement *"All women are polite. Irene is a woman. Therefore, Irene is polite"*
(c) *"If Urmy won the competition, then either Sreya came second or Samrat came third. Samrat didn't come third. Thus, if Sreya didn't come second, then Urmy didn't win the competition."*

35. (a) Show that $(p \vee q) \rightarrow q$ is logically equivalent to it is contrapositive using a truth table.
(b) Given that $S_1 = (p \rightarrow q) \rightarrow (\neg q \rightarrow \neg p)$ and $S_2 = \neg(\neg q \rightarrow \neg p) \rightarrow \neg(p \rightarrow q)$, use the truth table to show whether S_1 is equivalent to S_2
(c) Draw a simple network for $q \vee (p \wedge \neg q) \vee (r \wedge \neg p)$
(d) Construct a compound sentence $X(p,q,r)$ having the truth table shown below: -

p	T	T	T	T	F	F	F	F
q	T	T	F	F	T	T	F	F
r	T	F	T	F	T	F	T	F

$X(p,q,r)$	T	F	T	F	F	F	F	F

36. (a) Differentiate between Tautology and contradiction and give one example for each.
    (b) Determine whether or not, the statement $[(p \vee q) \wedge r] \rightarrow (p \wedge q)$ is a tautology.
    (c) Given that proposition $(p \wedge q) \rightarrow \neg r$
      Write: -
      (i) The converse of its contrapositive
      (ii) Inverse of its converse
    (d) Let *p* be Sam is rich and *q* be Sam is unhappy. Write each of the following in symbolic form.
      (i) Sam is poor but unhappy
      (ii) Sam is poor or else he is both rich and unhappy
      (iii) Sam is either rich or unhappy
      (iv) Sam is rich if and only if he is happy
37. (a) Show that $p \leftrightarrow q$ does not logically imply $p \rightarrow q$
    (b) What is meant by saying that an argument is valid?
    (c) Prove the validity of the following argument: *"If the country is democratic and governed by selfish leaders, then the country will develop. If the leaders are not selfish, the country will not develop therefore the country is not democratic."*
    (d) Construct a truth table and simplified logical circuit of: -
      (i) $p \wedge (p \rightarrow \neg q)$ (ii) $(p \rightarrow q) \rightarrow \neg p$
38. Consider two propositions *p* and *q*, complete the truth table below.

p	q	$\neg q$	$p \Rightarrow \neg q$	$\neg p$	$\neg p \Rightarrow q$

Decide whether the compound proposition $(p \Rightarrow \neg q) \Leftrightarrow (\neg p \Rightarrow q)$ is a tautology. State the reason for your decision.

39. Complete the truth table shown below.

p	q	$p \wedge q$	$p \vee (p \wedge q)$	$(p \vee (p \wedge q)) \Rightarrow p$
T	T			
T	F			
F	T			
F	F			

State whether the compound proposition $(p \vee (p \wedge q)) \Rightarrow p$ is a contradiction, a tautology or neither.

40. Consider the following propositions.

p: Isaac finishes her homework

q: Isaac goes to the football match

Write in symbolic form the following proposition. If Isaac does not go to the football match, then Isaac finishes his homework.

41. (a) Complete the following truth table.

p	q	....	$p \Rightarrow \neg q$
T	T	F	....
T	F	T	....
F	T	F	....
F	F	T	....

Consider the propositions

p: Cristina understands logic

q: Cristina will do well on the logic test.

(b) Write down the following compound proposition in symbolic form.
"If Cristina understands logic, then she will do well on the logic test"

(c) Write down in words the contrapositive of the proposition given in part (b).

42. Consider the two propositions *p* and *q*.

p: The sun is shining   *q: I will go swimming*

Write in words the compound propositions

(a) $p \Rightarrow q$;

(b) $\neg p \vee q$.

(c) The truth table for these compound propositions is given below.

p	q	K	$\neg p$	M
T	T	T	…………	T
T	F	F	…………	F
F	T	T	…………	T
F	F	T	…………	T

Complete the table

(d) State the relationship between the compound propositions *K* and M.

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Logic is the thrid topic for the advanced mathematics Tanzania syllabus. This book covers all necessary parts of the topic to help learners and facilitators. The exercise provided in this book has got the suggested answers at the end of the book. The writer also wrote other mathematics and ICT books from primary level to advanced level. To get other books visit www.jihudumie.com and navigate to the library.

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