THE EARTH AS A SPHERE

Tanzania syllabus

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THE EARTH AS A SPHERE

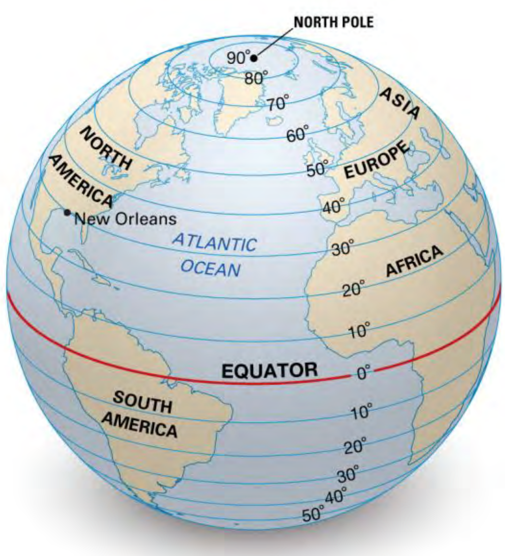
The Earth: This is one of the planets in our solar system. This is a planet we live on (up to now 2021)

The sphere: Is a solid object (3 dimensional) with round surface which is equidistant from the fixed point called **Center.**

The earth

LATITUDES: These are imaginary lines drawn on the surface of the earth running from East to west around the earth and they are measured in degrees North and South of the equator.

The latitude which divides the earth into two equal hemispheres is called the **EQUATOR.**



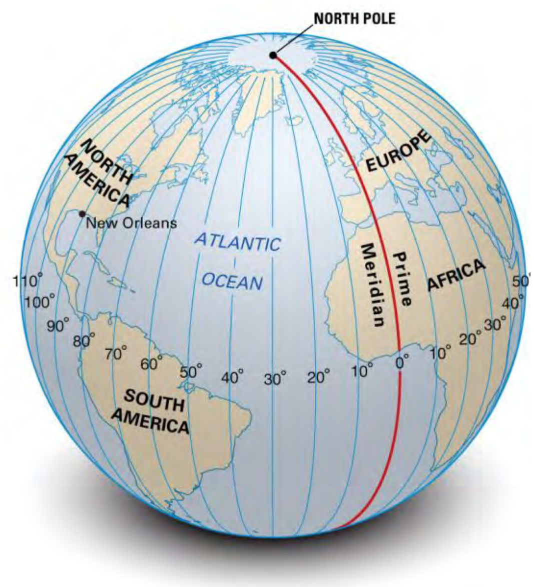
Facts about Latitudes

- They are all parallel, never meet
- Run in an east-west direction
- Cross the prime meridian at 90^o
- Get shorter toward the poles, with the only Equator the longest (great circle)

LONGITUDES:

These are imaginary lines drawn on the surface of the earth running from north pole to the south pole round the earth, and they are measured in degrees West and East of the prime meridian called **GREENWICH MERIDIAN.**

The name Greenwich meridian is given to this longitude because it passes through the Greenwich City in England



Facts about longitudes

- They are also called meridians
- Run in a north-south direction
- Measured in degrees East or west of the prime meridian
- Crosses the equator at 90^o
- They are all equal in length
- They are all great circle

The central angle may be from the parallel of latitudes or longitudes

If the central angle is from latitudes, it will lie on the same meridian. The rules like

- **SSS – Same Sign Subtract** and
- **DSA – Different Sign Add**

applied

Importance studying latitudes and longitudes

- When used together, longitude and latitude define a specific location through geographical coordinates. These coordinates are what the Global Position System (GPS) uses to provide an accurate locational relay.
- Used in Time and dates: **International Date Line:** The International Date Line (IDL) is on the opposite side of the earth to the Prime Meridian, located at 180 degrees. Together, the Prime Meridian and IDL divide the earth into two halves: the Western and Eastern Hemispheres. The IDL is the point at which the change of day takes place. When you travel from east to west across the IDL, you gain a day. Likewise, when you travel from east to west you lose a day. *Example*: Australians travelling to the USA often arrive at their destination before their departure time, because they've gained a day. When they go back to Australia however, they lose a day. IDL goes zig-zag in order to avoid the confusion of having different dates in the same country.
- Used in navigation: No roads in the oceans or seas. The navigators use Longitudes and latitudes to now their current position and their destination. The compass and the GPS devices work together to guide them to the destination. Same applies to airplanes.

Points on the surface of the earth

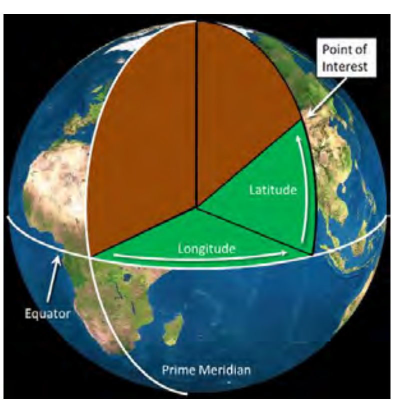
When using longitudes and latitudes to show the place on the earth surface it is always the latitude first and then longitude. The latitudes end with North (N) or South (S) while longitudes end with West (W) or East (E). *Example*: Arusha Tanzania is at (3.4°S, 36.7°E)

Central angle

A central angle is subtended on the great circle (meridian or longitude) if the latitude degree changes while the longitude degree is the same.

Points like P(X1°N/S, Y°E/W) and Q(X2°N/S, Y°E/W) is on the great circle.

Central angle is an angle made at the center of the earth.



Example 1

Calculate the central angle subtended by the following points on the earth surface.

A(30°N, 60°W) and B(70°N, 60°W)

Working;

Note that, these points lie on the same meridian 60°W, so they lie on the great circle.

The central angle = ϴ = 70° – 30° = 40°. The rule here is **SSS,** because A is 30° **North** and B is 70° **North too.** So, the **SSS is used to get the central angle**

Example 2

Calculate the size of the central angle subtended by the following points on the earth surface

C(50°N, 45°E) and D(35°S, 45°E)

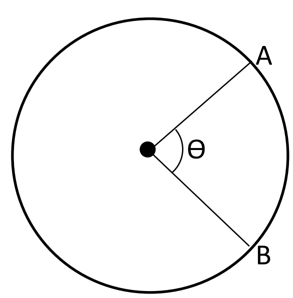
Working;

Note that, these points lie on the same meridian, 45°E, so, they lie on the great circle.

The central angle = ϴ = 50° + 35° = 85°. The **DSA** rule applies here because C is on latitude 50° **North** while D is on latitude 35° **South.** These are different sides, and the rule is **DSA.**

DISTANCE ALONG THE GREAT CIRCLE

The distance can be in km or nm. The great circles distance are all meridian including the equator.



Let the circle to the left be the meridian around the world. We are interested with the distance along this median which is the circumference of the circle.

The full circle has 360° while the subtended arc has ϴ degree.

- *Let the length AB be D*
- *The circumference be C*

By similarities concept;

$$\frac{C}{360^\circ}=\frac{D}{\theta} \text{ thus } D=\frac{C\theta}{360^\circ}$$

$$C=2\pi R$$

$$D=\frac{2\pi R\theta}{360^\circ} \text{ Simplifying,}$$

$$D_{km}=\frac{\pi R\theta}{180^\circ}km \qquad (i)$$

$$D_{nm}=60\times\theta nm \qquad (ii)$$

Example

Calculate the distance in kilometers from these two towns A(40°N, 70°W) and B(25°S, 70°W)

Working

Central angle = Ө = 40° + 25° = 65°

$$D=\frac{\pi R\theta}{180^\circ}$$

$$D=\frac{3.14\times6370\times65^\circ}{180^\circ}$$

Distance = 7222.9 km

Example 4

Find distance between two towns located on latitude 40°N and 60°S both lies on the median 103°W

Working

$$D_{km}=\frac{\pi R\theta}{180^\circ}km$$

$$D_{km}=\frac{3.14\times6370\times100^\circ}{180^\circ}=11112.1$$

The distance between the towns is 11112.1 km

Example 5

Find the distance in both km and nm between the following places

Morogoro (7°S, 38°E) and Moscow (56°N, 38°E)

In Km

$$D_{km}=\frac{\pi R\theta}{180^\circ}km$$

$$D_{km}=\frac{\pi\times6370\times63^\circ}{180^\circ}km=7004.2km$$

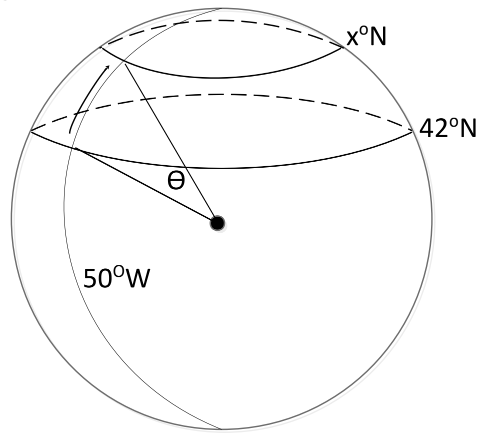
(b) $D_{nm}=60\theta$

$D_{nm}=60\times63^\circ=3780nm$

Example 6

A ship sails 5000km due North from (42°N, 50°W). Find it is new position for the journey.

Workings



Ө = x° – 42°

$$5000=\frac{\pi\times6370(x-42)^\circ}{180^\circ}km$$

45 = x – 42

X = 87°

The new position of the ship is (87°N, 50°W)

Example 7

Find the distance along a circle of longitude between A(28°N, 30°E) and B(12°S, 30°E) in both km and nm

<u>In km</u>

$D_{km}=\frac{\pi R\theta}{180^\circ}$, and Ө = 28° + 12° = 40°

$$D_{km}=\frac{3.14\times6370\times40^\circ}{180^\circ}=4447.1km$$

<u>In nm</u>

$D_{nm}=60\theta$

$D_{nm}=60\times40=2400\ nm$

Example 8

Find the length of the equator around the earth in Km and in Nm.

Working

Distance of the equator is the circumference of the circle with R = 6370km

<u>In km</u>

$D_{km} = 2\pi R$, Thus,

$D_{km} = 2 \times 3.14 \times 6370$ = 40023.89km

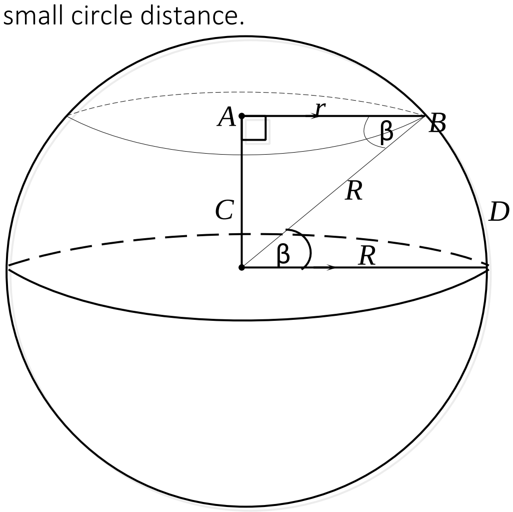
<u>In nm</u>

$D_{nm} = 60\theta$

$D_{nm} = 60 \times 360^{o} = 21600$nm

DISTANCE ALONG THE SMALL CIRCLE

The distance along the small circle is all distances involving latitudes excluding the equator. The distance along the latitudes is what we refer as the small circle distance.



Suppose, we want to find the length of the latitude ϴ North. (see the figure above)

AB is parallel to CD while CD and CB are both radii of the earth, therefore, angle ABC = angle BCD. Let AB be r (radius of the latitude), using cosine formula

$$\cos\beta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

$$\cos\beta = \frac{r}{R} \Rightarrow r = R\cos\beta$$

The length of the latitude is the perimeter of the circle around the earth with radius r.

Thus, the length of any latitude is given by $L = 2\pi r \Rightarrow 2\pi R\cos\beta$

Example 9

Find the length of the latitude 75°S in km.

Working

$L = 2\pi R\cos\beta$

$L = 2 \times 3.14 \times 6370 \cos(75^{o}) = 10359$ km

Example 10

Find the length of latitude 53°N in nm

Working

$D_{nm} = 60 \times 360^{o} \times \cos\beta$

$D_{nm} = 60 \times 360^{o} \times \cos(53^{o}) = 13000$nm

OR using conversion 1.852 km = 1 nm

$$D_{nm} = \frac{D_{km}}{1.852}\text{nm}$$

$$D_{nm} = \frac{2 \times 3.14 \times 6370\cos(53^{o})}{1.852} = 13000\,\text{nm}$$

DISTANCE ALONG TWO POINTS ON PARALLEL OF LATITUDE

The distance in km is given by

$$D_{km} = \frac{\pi R\theta\cos\beta}{180^{o}}$$

The distance in nm is given by

$$D_{nm} = 60\theta\cos(\beta)$$

In both cases *ϴ* is the degree change in longitude while β is the degree of the latitude in concern.

Example 11

Find the distance in both km and nm from the following points A(80°N, 30°E) and B(80°N, 35°W)

Working

For this case,

$\theta = 30^{o} + 35^{o} = 65^{o}$

$\beta = 80^{o}$ (degree of latitude)

$$D_{km} = \frac{\pi \times 6370 \times 65^{o} \times \cos(80^{o})}{180^{o}} = 1255 \text{ km}$$

and

$$D_{nm} = 60 \times 65^{o} \times \cos(80^{o}) = 677 \text{ nm}$$

Example 12

Find the distance along a circle of latitude between M(30°N, 48°W) and N(30°N, 79°W) in both km and nm

Working

This is the distance along the small circle

$\theta = 79^\circ - 48^\circ = 31^\circ$ and $\beta = 30^\circ$

$$D_{km} = \frac{\pi R\theta\cos\beta}{180^\circ}$$

$$D_{km} = \frac{\pi \times 6370 \times 31^\circ \times \cos 30^\circ}{180^\circ}$$

$D_{km} = 2985$ km

$D_{nm} = 60\theta\cos\beta$, thus $D_{nm} = 60 \times 31^\circ \times \cos 30^\circ$

$D_{nm} = 1611$ nm

Example 13

Points A and B both lies on the same latitude 36°S of the equator. The longitude of A is 40°W. If the distance from A to B is 194.2 nautical miles, find the degree of longitude of B correct to nearest whole number.

Working

$D_{nm} = 60\theta\cos\beta$

Given $\beta = 36^\circ$ while θ = ?

$194.2 = 60 \times \theta \times \cos(36^\circ)$

$\theta = 4^\circ$

But $\theta = 40^\circ \pm \alpha$ where α is the degree of longitude of B

- Let assume B is in the west of A
  $\theta = \alpha - 40^\circ$ thus $\alpha = 4^\circ + 40^\circ = 44^\circ$ the degree of longitude of B is 44°
- Again, let assume B to be in the east of A, thus
  $\alpha = 40^\circ - 4^\circ = 36^\circ$
  The longitude of B may be 44°W and 36°W

Example 14

A ship starts its journey at (40°N, 19°W) and sails due East 3300 nm. Find the location of its new place.

Working

$D_{nm} = 60\theta\cos\beta$

$3300 = 60 \times \theta \times \cos 40^\circ$

$$\theta = \frac{3300}{60\cos(40^\circ)} = 71.8^\circ$$

But $\theta = 19^\circ + \alpha$

$71.8^\circ = 19^\circ + \alpha$

$\alpha = 71.8^\circ - 19^\circ = 52.8^\circ$

The position of a ship is (40°N, 52°E)

MISCELLANEOUS EXAMPLES

Example 15

An airbus started a journey at Tropic of Cancer 100°E to Tropic of Capricorn while maintaining the longitudes. If the journey started at 10:15 pm on Tuesday at the speed of 792 knots. When and at what time will it reach the destiny?

Working

T. Cancer (23.5°N, 100°E) and T. Capricorn (23.5°S, 100°E)

$D_{nm} = 60\theta$

$\theta = 23.5^\circ + 23.5^\circ = 47^\circ$

$Dnm = 60 \times 47^\circ = 2820$ nm

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$792 = \frac{2820}{T} \text{ thus } T = \frac{2820}{792} = 3.56$$

It took 3 hours, 33 minutes and 36 second to reach the destination.

The airbus will reach the destination at 1:33:36 on the next day, Wednesday.

Question: *Repeat example 15 above if the airbus started at Antarctic circle to the Arctic circle on longitude 33°W.*

Example 16

Three towns X, Y and Z are on the same latitude, 31°N with town Y lie East of X while West of Z. If Y is on prime meridian and the distance between XY is 7200nm while the distance between XZ is 11400nm. Find the positions of X, Y and Z.

Working

Distance XY = 7200nm

The longitude of Y = 0°

$7200 = 60(x - 0)$

The longitude of X is 120°. The position of X is (31°N, 120°W). The position of Y is given (31°N, 0°)

The distance YZ = XZ – XY = 11400 – 7200 = 4200 nm

$4200 = 60(y - 0)$ Thus Y = 70°E

The position of Z is (31°N, 70°E)

OTHER MATHEMATICS TOPIC NOTES AVAILABLE

FOR O-LEVEL

1) All form 1 Mathematics + ICT topics
2) All form 2 mathematics + ICT topics
3) All form 3 mathematics + ICT topics
4) All form 4 mathematics + ICT topics
5) All additional mathematics topics

FOR A-LEVEL

1) All form 5 advance mathematics topics
2) All form 5 BAM topics
3) All form 6 advance mathematics topics
4) All form 6 BAM topics

PHYSICS NOTES AVAILABLE

FOR O-LEVEL

1) Physics form 1 full notes
2) Physics form 2 full notes
3) Physics form 3 full notes
4) Physics form 4 full notes

PHYSICS + ICT FOR A-LEVEL

**Preparation in progress until January 2022*

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